Flatness Error Evaluation of the Satellite Antenna

Tengda Liu, Chunjie Wang, and Bingjie Wang

***Abstract*—Synthetic aperture radar (SAR) antenna is suitable for small satellite platform. It deforms easily in unfolded state by the affection of the space thermal loads on orbit, which directly reduces the precision of the antenna. The flatness error is an important index to evaluate the thermal deformation of antenna surface. In this paper, first a new flatness error evaluation algorithm of plane in any space placement based on least square method was proposed for accurate assessment of the satellite antenna thermal deformation degree. Then the finite element model of SAR antenna for thermal deformation analysis was established and the measure points of the SAR antenna's measure surface was picked. Finally the antenna plate flatness error was calculated based on the new evaluation algorithm and the result of analysis. The flatness error of the satellite antenna plate quantifies the degree of the satellite antenna deformation and provides the basis for further design optimization of the SAR antenna structure.**

***Index Terms*—Flatness error evaluation, least square method, satellite antenna, thermal deformation analysis.**

I. Introduction

Synthetic aperture radar (SAR) satellite is the new earth observation system. The antenna of the satellite is the important payload [1]. The SAR antenna no matter the active phased planar array or the mesh reflector antenna has a large size. When it is exposed in both solar radiation and dark space, the temperature gradient along spanwise direction and thickness direction causes thermal deformation in antenna plate and bracing truss [2]. The reflector requires high geometric precision, so the analysis of the temperature field and the antenna surface thermal deformation is necessary. Based on the data of thermal deformation, the surface displacement of the SAR antenna can be controlled and corrected. The flatness of satellite antenna which is a reflection of the antenna precision is a key technical index to measure the deformation degree [3]. Furthermore, how to evaluate the flatness of satellite antenna is an important index to evaluate the design of structure of satellite antenna.

Least square method is one of the most commonly used methods to evaluate flatness error of the measure surface. But to solve least square plane, a specific component of the normal vector of fitting plane has to be provided at first [4]. This is quite difficult for fitting the plane of SAR antenna due to its spatial structure, especially for the mesh reflector antenna. In this paper, a new flatness error evaluation algorithm of plane in any space placement is proposed to measure the flatness error of SAR antenna after the thermal deformation. This method provides the basis of further optimization design for the design of SAR antenna structure.

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II. The Principle of the Flatness Error Evaluation Algorithm

This paper uses least square method to assess flatness error of plane in any space placement. Flatness error is the variation quantity between the real surface being measured and its ideal plane, and the ideal plane can be replaced by least square plane in practice. When using least square method to evaluate the flatness error, the datum plane to be evaluated is the least square plane. As is shown in Fig. 1, a set of measure points as $P(x_i, y_i, z_i)$ are picked from the measure surface. By least square method, a new plane which can be expressed as an equation $ax+by+cz+d=0$. That plane is the least square plane for the measure surface [5]. There is no need to do many trial calculations to get the accurate flatness error with least square method. To evaluate flatness error by least square method, the first step is to fit the least square plane of the measure point. Usually this method needs value of a specific component of the normal vector of fitting plane according to the spatial distribution of the measurement point [6]. As there are limitations in the existing method, a new solving method is demanded.

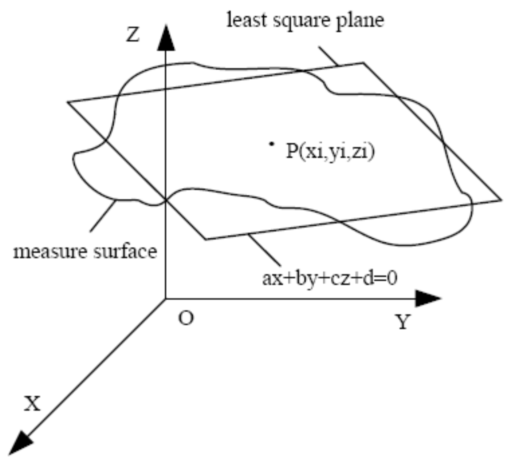

Fig. 1. Space relation between least square plane and the measure surface.

A. Fitting Algorithm of Plane in Any Space Placement

To obtain the fitting plane, suppose (x_i, y_i, z_i) $(i=1,2,...,k)$ are k measure points and the equation of the to-be-fitted plane S_{LS} is:

$$ax+by+cz+d=0 \tag{1}$$

where (a,b,c) is the unit normal vector of S_{LS}, and for simplicity, it is assumed that:

$$a^2+b^2+c^2=1 \tag{2}$$

According to least square principle, S_{LS} should have the smallest sum of square of the distance from all measure points to it:

$$\min : \mathrm{L}(a,b,c,d)=\sum_{i=1}^{k} d_i^2 \tag{3}$$

where d_i is the distance between the measure point (x_i, y_i, z_i) and the plane S_{LS}, and it can be expressed as:

$$d_i=\left|ax_i+by_i+cz_i+d\right| \tag{4}$$

In order to make $\mathrm{L}(a,b,c,d)$ to achieve its minimum, the following equation has to be satisfied:

$$\frac{\partial L}{\partial d}=2a\sum_{i=1}^{k}x_i+2b\sum_{i=1}^{k}y_i+2c\sum_{i=1}^{k}z_i+2d=0 \tag{5}$$

Therefore, the value of d is obtained:

$$d=-(\bar{x}a+\bar{y}b+\bar{z}c) \tag{6}$$

where:

$$\begin{aligned} \bar{x}&=\sum_{i=1}^{n}x_i/n, \\ \bar{y}&=\sum_{i=1}^{n}y_i/n, \\ \bar{z}&=\sum_{i=1}^{n}z_i/n \end{aligned} \tag{7}$$

Put (6) into (1), the following equation is obtained:

$$\begin{aligned} \sum_{i=1}^{n}d_i^2 &= D_{11}a^2+D_{22}b^2+D_{23}c^2 \\ &+(D_{12}+D_{21})ab+(D_{13}+D_{31})ac \\ &+(D_{23}+D_{32})bc \end{aligned} \tag{8}$$

where the specific expression of the parameters ($D_{11} \sim D_{33}$) is:

$$\begin{aligned} D_{11}&=\sum_{i=1}^{n}x_i^2-n\bar{x}^2, \\ D_{22}&=\sum_{i=1}^{n}y_i^2-n\bar{y}^2, \\ D_{33}&=\sum_{i=1}^{n}z_i^2-n\bar{z}^2, \\ D_{12}=D_{21}&=\sum_{i=1}^{n}x_iy_i-n\overline{xy}, \\ D_{13}=D_{31}&=\sum_{i=1}^{n}x_iz_i-n\overline{xz}, \\ D_{23}=D_{32}&=\sum_{i=1}^{n}y_iz_i-n\overline{yz} \end{aligned} \tag{9}$$

Introducing a 3×3 matrix:

$$D=\begin{bmatrix} D_{11} & D_{12} & D_{13} \\ D_{21} & D_{22} & D_{23} \\ D_{31} & D_{32} & D_{33} \end{bmatrix} \tag{10}$$

Then the equation (8) can be rewritten as:

$$\begin{aligned} \sum_{i=1}^{n}d_i^2 &= D_{11}a^2+D_{22}b^2+D_{23}c^2+2D_{12}ab \\ &+2D_{13}ac+2D_{23}bc \\ &=[a,b,c]D[a,b,c]^T \end{aligned} \tag{11}$$

Obviously D is a symmetric matrix. If $\lambda_{\min}(D)$ is the smallest eigenvalue of the matrix D, the following inequality holds:

$$\sum_{i=1}^{n}d_i^2 \geq \lambda_{\min}(D) \tag{12}$$

When $[a,b,c]^T$ is the corresponding units eigenvector of $\lambda_{\min}(D)$, the function (3) can get its minimum value. We can get the equation of the fitting plane of the measure points simply by solving the units eigenvector of the smallest eigenvalue of the matrix D.

B. Flatness Error Calculation of Plane in Any Space Placement

The maximum distance $d_{\max}$ and minimum distance $d_{\min}$ between the measure points and S_{LS} which is used as evaluation datum plane can be calculated, and the flatness error is $f_{LS}=d_{\max}-d_{\min}$. Based on the deduction above, as shown in Fig. 2, the process of solving flatness error of fitting plane with the new method can be obtained.

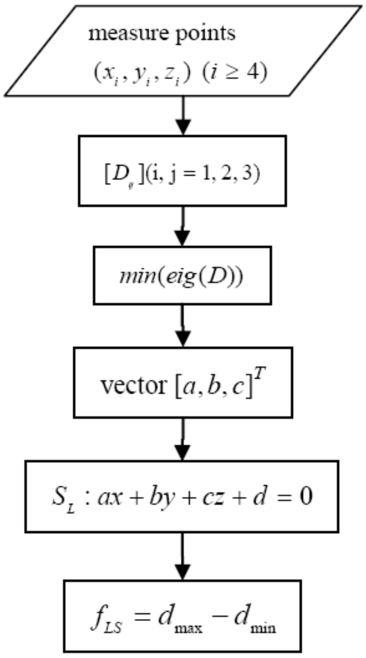

Fig. 2. Process of solving flatness error of fitting plane with new method.

The value of a specific component of the normal vector of fitting plane according to the spatial distribution of the measure point doesn't need to be given beforehand in the new algorithm, therefore its generality is more stronger and it is easier for programming.

III. Finite Element Model of SAR Satellite Antenna

Composite materials are adopted in SAR satellite antenna whose surface is working cells as substrate. On the back of the antenna, there is a set of foldable space truss structure in order to ensure the strength and stability of antenna when spreading. The calculation and measurement of antenna thermal deformation are very important during the design of the satellite antenna. So a lot of calculation and measurement method are raised by numerous scholars such as Electronic Speckle Pattern Interferometry (ESPI) [7], Digital Close Range Photogrammetry, and Strain Real-time Measurement [8]. The measurement analysis of antenna thermal deformation by simulating space thermal environment can get the satellite antenna deformation effectively. But this method has the disadvantages of higher cost and longer period. The design of the satellite antenna needs to be modified repeatedly which obviously adds the design cost and period. Thus it is important to use digital simulation technology to analysis and forecast the antenna thermal deformation. The finite element model of SAR satellite antenna in the unfolded state is built as shown in Fig. 3.

The antenna substrate which is simulated by shell element is fixed on the antenna frame. The space truss structure is composed of the rod and the hinge. The model can be simplified by using beam element to simulate rods between which the hinge and its rigidity of six directions are simulated by spring damping element. The connections of hinge and rod are simulated by rigid element. Then the simulation analysis of the satellite antenna thermal deformation is completed after simulating the space thermal environment by setting thermal load parameters.

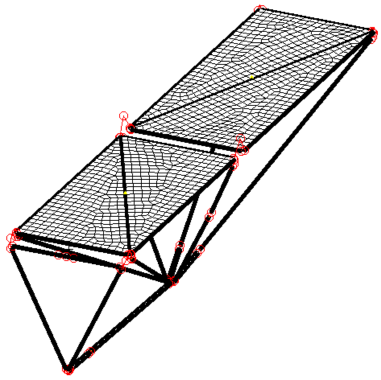

Fig. 3. The finite element analysis model of the SAR satellite antenna.

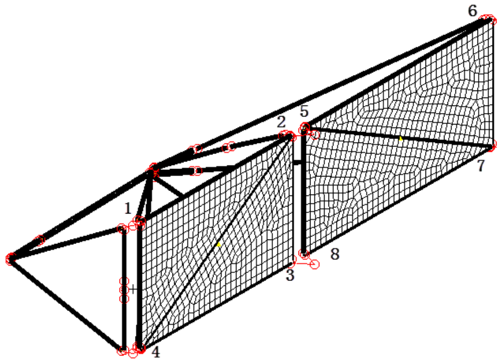

Fig. 4. The position of 8 measure points on antenna panels.

IV. Evaluation of SAR Satellite Antenna Flatness Error

The thermal deformation of SAR satellite antenna directly influences the antenna working accuracy. The coordinate position of the measurement points selected on the antenna panel are measured after the thermal deformation. Based on the algorithm above, the flatness error of antenna after thermal deformation can be evaluated. The flatness error of the antenna panel can quantify the degree of the satellite antenna thermal deformation, and to evaluate the influence of antenna working accuracy with antenna thermal deformation. Meanwhile, it can be taken as basis of further optimization design of the antenna structure.

The thermal deformation of the satellite antenna bracing truss would cause global deformation which affects the working accuracy of the global satellite antenna array elements. 8 measure points are chosen from 2 antenna panels, the fitting points distributed in the connected vertex between framework and panel are used to evaluate the global deformation degree [9], as shown in Fig. 4 and Fig. 5.

Meanwhile the partial thermal deformation due to the heating transfer disproportionation also affects the working accuracy of partial satellite antenna array elements. So another 4 measurement points are picked from partial large thermal deformation region. Together those measurement points can evaluate the degree of satellite antenna local deformation.

By putting the coordinate value of the measurement points into formula (7), the fitting plane equation $ax + by + cz + d = 0$ can be solved. The distance between each measurement point and the fitting plane is $d_i = ax_i + by_i + cz_i + d$, the maximum value is $d_{\max}$, the minimum value is $d_{\min}$. Thus the flatness error of the satellite antenna panel is f_{LS}.

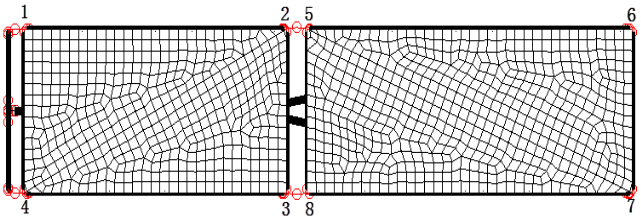

Fig. 5. The vertical view of antenna panels with 8 measure points.

As the process shown in Fig. 2, the coordinate value of the 8 measure points are needed. Their three coordinates value under global coordinate system are listed in Table I.

TABLE I: THE COORDINATE VALUE OF MEASURE POINTS

Measure points	X (mm)	Y(mm)	Z(mm)
1	1082.310	-2790.201	-209.801
2	569.658	-1437.329	-112.669
3	375.204	-1474.172	-625.826
4	887.856	-2827.040	-722.958
5	551.977	-1390.671	-109.319
6	21.651	8.839	-8.839
7	-172.804	-28.004	-521.996
8	357.523	-1427.509	-622.476

Finally, two fitting plane equations of SAR antenna are acquired:

$$\begin{aligned} 8.6602541x + 3.5355333y - 3.5355342z = 250.023 \\ 8.6602539x + 3.5355344y - 3.5355337z = 250.011 \end{aligned} \quad (13)$$

Based on the equations of fitting plane, the flatness error of the satellite antenna panel is $1.49\mu m$ and $1.44\mu m$.

V. CONCLUSION

In this paper, the flatness error evaluation algorithm of plane in any space placement based on least square method is proposed. The innovation of this method lies on the point that there is no need to provide a specific component of the normal vector of the fitting plane. It is more suitable for multi planes with multi angles such as SAR antenna. The generality of this method is stronger and it is easier for programming.

Based on the finite element model of SAR satellite antenna, the analysis of structure thermal deformation is completed. It gives out a new method of evaluating degree of satellite antenna thermal deformation based on the flatness error evaluation algorithm proposed in this article. This method quantifies degree of satellite antenna thermal deformation, and provides effective proof for the further optimization design of the antenna structure which obviously improve the working efficiency in the period of SAR antenna structure design.

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