PAPR Reduction Based on DFT Precoding for OFDM Signals

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***Abstract*—The major problem of Orthogonal Frequency Division Multiplexing (OFDM) is its High Peak-to-Average Power Ratio (PAPR). The High PAPR increases the complexity of Analogue to Digital (A/D) and Digital to Analogue (D/A) converters and also reduces the efficiency of Radio Frequency High Power Amplifier (RF HPA). In this paper, we present Discrete Fourier Transform (DFT), Discrete Hartley Transform (DHT) and Zadoff-Chu Transform (ZCT) precoders for both clipping and clipping and filtering to reduce PAPR. The DFT precoder provides better PAPR compared with clipping, clipping and filtering OFDM. In addition to improve the bit error rate (BER) this has also been taken as a performance evaluation parameter. The DFT precoded system is better than ZCT precoder by about 1.2 dB and better than DHT precoder by about 1.5 dB at the same Complementary Cumulative Distribution Function (CCDF) value. The DFT precoder is better than clipping by 1dB and better than clipping and filtering technique by about 6dB for OFDM system at the same parameters. The DFT Precoded system is better than other precoder as well as it improvement in BER performance of the original OFDM by about 1 dB at the same BER.**

***Index Terms*—OFDM, DFT, ZCT, DHT.**

I. Introduction

OFDM is a multicarrier transmission scheme that has become the technology of choice for next generation wireless and wire line digital communication systems because of its high speed data rates, high spectral efficiency, high quality service and robustness against narrowband interference and frequency selective fading. Due to its favorable features, OFDM has been adopted as a major data transmission technique by many wireless communication standards, such as IEEE 802.11a [1], IEEE 802.16a [2] and terrestrial digital video broadcasting (DVB-T) [3] systems in Europe.

An OFDM transmitter can be implemented by using inverse fast Fourier transform (IFFT), the output of which is a time domain signal. The outputs of IFFT (OFDM signals) have an inherent difficulty that it may exhibit very high peaks since it is generated by the addition of several independently modulated signals. The power of these large peaks will be very high compared to the average power of the signal. Hence peak to average power ratio is very high which is considered as the major disadvantage of the OFDM technique. These large peaks cause saturation in power amplifiers which is placed at the front end of the transmitter and leads to nonlinear distortions [4].

Several techniques have been proposed in the literature to reduce the PAPR. These techniques are constellation shaping [5], phase optimization [6], nonlinear companding transforms [7], Tone Reservation (TR) and Tone Injection (TI) [8-9], clipping and filtering [10], Partial Transmit Sequence (PTS) [11], precoding based techniques [12-13] and Precoding based Selected Mapping (PSLM) [14] are popular. The precoding based techniques, however, show great promise as they are simple linear techniques to implement without the need of any side information. Precoding improves PAPR without increasing much complexity and without destroying the orthogonality between subcarriers. The precoding also improves the BER in comparison to normal OFDM system because of diversity gain obtained due to the spreading of data symbol among more than one subcarrier. In this paper, we present DFT precoder for clipping, clipping and filtering to reduce PAPR.A performance comparison is made with other transform-based precoders, namely DHT and ZCT [12-13].

The paper is organized as follows. PAPR Based on Clipped OFDM signals is given in section II. Proposed Precoded Clipped OFDM Signals and its simulation results are given in section III, IV. Finally, conclusions are given in section V.

II. Clipped OFDM Signals

The clipping approach is the simplest PAPR reduction scheme, which limits the maximum of transmit signal to a pre-specified level. However, it causes in-band signal distortion; resulting in BER performance degradation also causes out-of-band radiation, which imposes out-of-band interference signals to adjacent channels. Although the out-of-band signals caused by clipping can be reduced by filtering which also improve BER Performance, this result in degradation in the PAPR. In fig.1 the entire bit stream that we have to transmit is divided into several blocks of N symbols each. For a data block $\mathbf{x} = [x_0, x_1, \cdots, x_{N-1}]$, the OFDM signal can be represented as [15]:

$$x_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k . e^{j2\pi \frac{n}{N} k} \quad , n = 0,1,2, \ldots N-1 \qquad (1)$$

The PAPR of OFDM signal in (1) can be written as:

$$PAPR = \frac{max|x_n|^2}{E[|x_n|^2]} \qquad (2)$$

where E denotes expectation, The CCDF for an OFDM signal can be written as:

$$P(PAPR > PAPR_0) = 1 - (1 - e^{-PAPR_0})^N \qquad (3)$$

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where $PAPR_0$ is the clipping level. This equation can be read as the probability that the PAPR of a symbol block exceeds some clip level $PAPR_0$. The clipping operation is defined as [16]:

$$|x(t)| = \begin{cases} |x(t)|, & if\ |x(t)| < A \\ A, & if\ |x(t)| \geq A \end{cases} \quad (4)$$

where A is a preset clipping level. The normalized clipping level is called clipping ratio and is defined as:

$$CR = \frac{A}{\max(x(t))} \quad (5)$$

When we decrease CR from 1 to 0.9, 0.8…. clipping increases.

III. Proposed DFT Precoded Clipped OFDM System

In the proposed system, The DFT precoder of the same size as IFFT is used as a "spreading" code. Thus, the OFDM system becomes equivalent to the Single Carrier (SC) system because the DFT and IDFT operations virtually cancel each other [17]. In this case, the transmit signal will have the same PAPR as in a single-carrier system which results in improvement in PAPR. Fig. 1 shows the block diagram of the proposed Clipped OFDM signals based on DFT Precoded.

We implemented the Precoding matrix P of dimension $N \times N$ before the IFFT to reduce the PAPR. The precoding matrix P can be written as:

$$P = \begin{bmatrix} p_{00} & p_{01} & \cdots & p_{0(N-1)} \\ p_{10} & p_{11} & \cdots & p_{1(N-1)} \\ \vdots & \vdots & \ddots & \vdots \\ p_{(N-1)0} & p_{(N-1)0} & \cdots & p_{(N-1)(N-1)} \end{bmatrix} \quad (6)$$

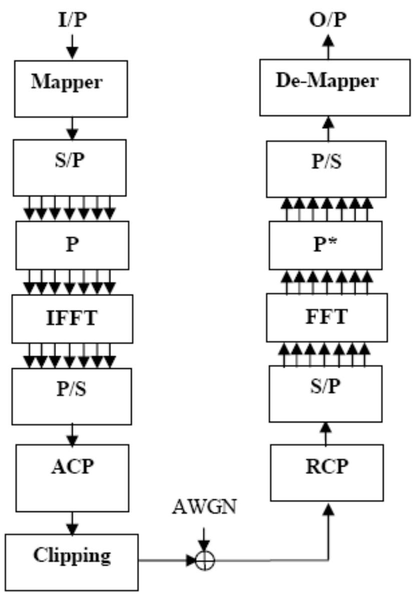

Fig. 1. Generic precoded clipped OFDM system block diagram

The complex base band OFDM signal with N subcarriers can be written as:

$$x(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} P.X_k.e^{j2\pi k\Delta f t} \quad 0 \leq t \leq NT \quad (7)$$

We can express modulated OFDM vector signal with N subcarriers as:

$$x_N = IFFT\{P.X_N\} \quad (8)$$

The PAPR of OFDM signal in (7) can be written as:

$$PAPR = \frac{max|x(t)|^2}{E[|x(t)|^2]} \quad (9)$$

The DFT of a sequence of length N and $IDFT$ can be written as:

$$X(k) = \sum_{n=0}^{N-1} x(n).e^{-j2\pi nk}, k = 0,1 \ldots .N-1 \quad (10)$$

$$x(n) = \frac{1}{N} \sum_{n=0}^{N-1} X(k).e^{j2\pi nk}, k = 0,1 \ldots .N-1 \quad (11)$$

where, $p_{mn} = e^{-j2\pi mn/N}$, m and n are integers from 0 to N-1.

IV. Simulation Results

We performed extensive simulations in MATLAB in order to evaluate the performance of DFT, DHT, and ZCT Precoder for both clipping and clipping and filtering OFDM system. To show the PAPR analysis of the proposed system, data is generated randomly

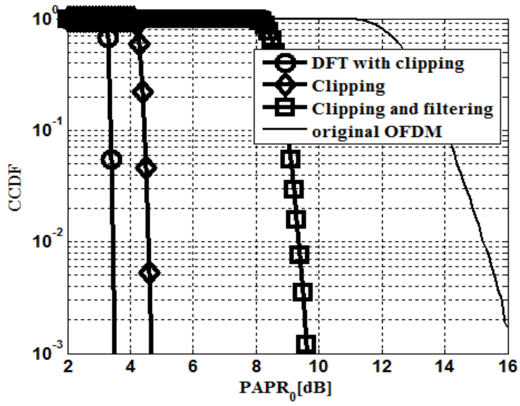

Fig. 2. CCDF comparison of DFT Precoding with clipping and clipping and filtering for QPSK OFDM system

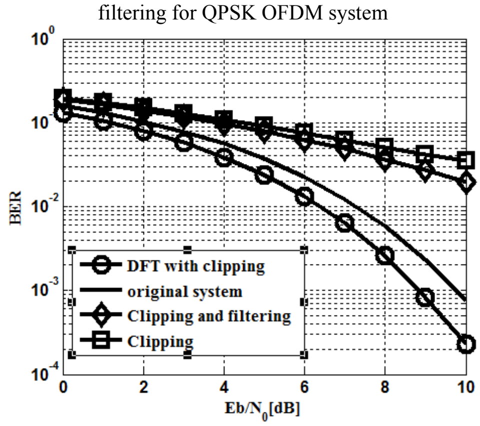

Fig. 3. BER for Clipped QPSK-OFDM with DFT precoding (N=128, CR=0.9)

Then modulated by QPSK. We compared the simulation results with clipping OFDM system. We considered FFT for N=128, number of Cyclic Prefix (CP) = 1/8, Clipping Ratio (CR) = 0.9, Over sampling factor S =8.

Fig. 2. Shows the CCDF comparisons of proposed technique with clipping and clipping and filtering for QPSK OFDM system and original QPSK OFDM system. The proposed system is better than clipping technique by 1dB

and clipping and filtering technique by more than 5dB.

As shown in Fig. 3, the proposed technique enhances the performance of BER more than Clipping technique and the original OFDM system, but this increase the processing time approximately by 30% in computer simulation results

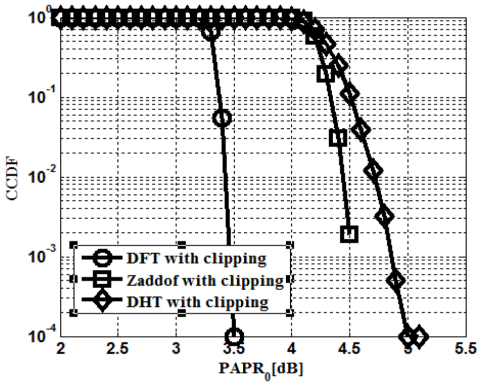

Fig. 4. CCDF comparison for DFT, DHT and ZCT Precoded clipped QPSK OFDM system

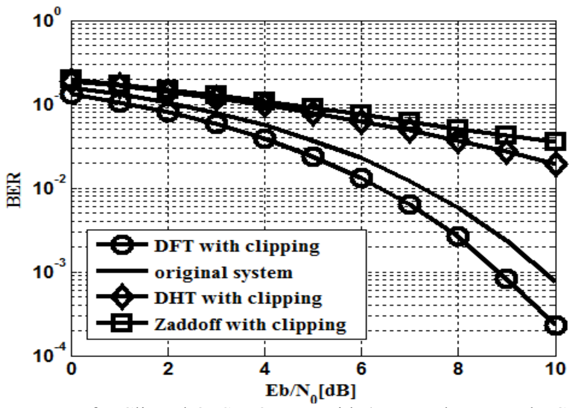

Fig. 5. BER for Clipped QPSK-OFDM with (DFT and DHT and ZCT) precoding (*N*=128, *CR*=0.9)

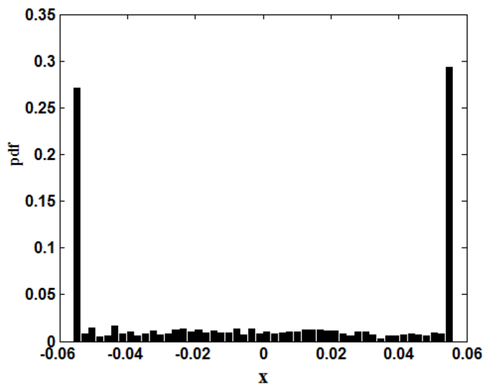

Fig. 6. (a) PDF of pass band clipped OFDM signal

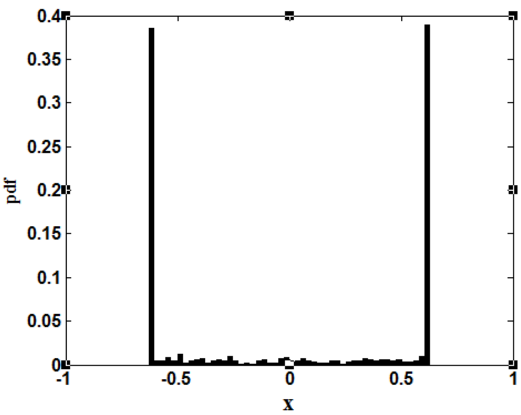

Fig. 6. (b) PDF of DFT precoded clipped OFDM signal

Fig. 4. Shows the CCDF comparisons of proposed technique with clipped signal based on DHT and ZCT precoder. The DFT Precoded system is better than ZCT precoder by about 1.2 dB and better than DHT precoder by about 1.5 dB at the same CCDF.

Fig. 5. Shows the BER comparisons of DFT, DHT and ZCT Precoded with clipped signal based on DHT and ZCT precoder. The DFT Precoded system is better than other precoder as well as it improvement in BER performance of the original QPSK OFDM by about 1 dB at the same BER.

Fig. 6(a), (b) shows the histograms as probability density functions (PDFs) of the passband clipped OFDM signal and the proposed DFT precoded clipped OFDM signal. We considered QPSK for *N*=128, *CP*=1/4, *CR*=0.8, Over sampling factor *S*=8.

As shown in Fig. 6(a) that x-axis ranged from $-0.06<x<0.06$.Fig. 6(b) precoding technique spread the PDF where the x-axis ranged from $-0.7<x<0.7$. which improve the performance of the proposed DFT precoded clipped OFDM than the clipped OFDM in both CCDF and BER as discussed in this paper.

V. Conclusions

In this paper, PAPR reduction scheme based on DFT, DHT and ZCT precoding schemes has been applied to the clipped OFDM signals. The clipping is the simplest PAPR reduction but resulting in BER Performance Degradation using filtering with clipping improve BER but effect on PAPR reversely using DFT precoder reduce this effect and improve BER than clipping technique, clipping and filtering and original OFDM. The DFT precoded system is better than ZCT precoder by about 1.2 dB and better than DHT precoder by about 1.5 dB at the same CCDF value. The DFT precoder is better than clipping by 1dB and better than clipping and filtering technique by about 6dB for OFDM system at the same parameters. The DFT Precoded system is better than other precoder as well as it improvement in BER performance of the original OFDM by about 1 dB at the same BER.

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